

Matrices

1. For each of the following matrices, which are representative of systems of linear equations you've already seen:

- What are the dimensions of the matrix?
- Is it a square matrix?
- What is the $(2,1)^{\text{st}}$ entry?
- What is the $(1, 3)^{\text{rd}}$ entry?
- What are the diagonal entries?
- What entries are on the super-diagonal?
- What entries are on the sub-diagonal?

$$\begin{pmatrix} 2 & -2 & 0 \\ 4 & -3 & -1 \\ 2 & -1 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & 4 & -3 & 0 & -2 \\ 1 & 2 & -2 & 1 & 0 \\ 3 & 2 & -2 & -2 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 0 & -1 & 0 \\ 4 & 6 & 0 & 0 & -2 \\ 0 & 0 & 2 & -2 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & -6 & 0 \\ 0 & 2 & -12 & 0 \\ 2 & 1 & -6 & -2 \end{pmatrix}$$

2. Given this matrix here, which is used to determine the amplitude and phase shift of a periodic function that is sampled periodically, what notation is used to represent the two columns vectors, and what notation is used to represent the entries?

$$V = \begin{pmatrix} 0 & 1 \\ 0.8414709848 & 0.5403023059 \\ 0.9092974268 & -0.4161468365 \\ 0.1411200081 & -0.9899924966 \\ -0.7568024953 & -0.6536436209 \\ -0.9589242747 & 0.2836621855 \\ -0.2794154982 & 0.9601702867 \end{pmatrix}$$

3. Suppose you had an infinite sequence of matrices:

$$A_1 = \begin{pmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{pmatrix}, A_2 = \begin{pmatrix} 8 & 9 \\ 10 & 11 \\ 12 & 13 \\ 14 & 15 \end{pmatrix}, A_3 = \begin{pmatrix} 16 & 17 \\ 18 & 19 \\ 20 & 21 \\ 22 & 23 \end{pmatrix}, \dots$$

Consider (consider only) the following questions:

- How would you refer to the 17th matrix in this sequence?
- How would you refer to the vector corresponding to the 1st column of the 3rd matrix A_3 ?
- How would you refer to the vector corresponding to the 2nd column of the 39th matrix A_{39} ?
- How would you refer to third entry of the first column of the 5th matrix A_5 ?
- How would you refer to entry $(4, 1)$ of the second matrix A_2 , and what is that value?

For the first part, the answer is clearly A_{17} . One approach would be to separate the sequence identifier from the matrix information by a comma, and another may be to use a semi-colon. For example, $\mathbf{a}_{3,1}$ vs. $\mathbf{a}_{3;1}$, $\mathbf{a}_{39,2}$ vs. $\mathbf{a}_{39;2}$, $a_{5,3,1}$ vs. $\mathbf{a}_{5;3,1}$, and $a_{2,4,1}$ vs. $a_{2;4,1}$. Which do you prefer? Which notation is less likely to cause confusion? Any other suggestions?